Formale Entwicklung objektorientierter Software

Praktikum im Wintersemester 2008/2009

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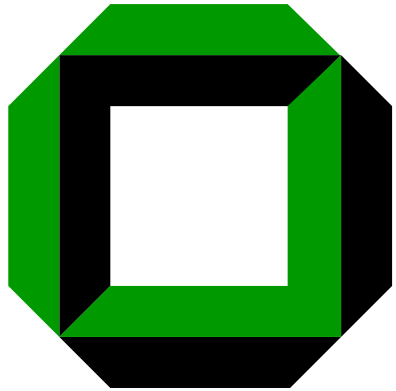

Loop Invariants

Unwinding Loops

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- 0 iterations?

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We need an **invariant rule** (or some other form of induction)

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Idea behind loop invariants

- Find a formula Inv which...
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Precondition: $\texttt{a} \doteq \texttt{null}$ & $\mathit{ClassInv}$

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& $\mathit{ClassInv}'$

Basic Invariant Rule: Problem

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- Required context information must be added to loop invariant Inv

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- Solution: **anonymising updates** $\mathcal{V}$ delete information about modified locations

$$\mathcal{V} = \{\mathtt{i} := i_0 \mid \mathtt{for}\ x;\ \mathtt{a}[x] := arr_0(\mathtt{a}, x)\}$$

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  - **Avoid this**! Always give a more narrow `assignable` clause.

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$\rightsquigarrow$ The invariant rule in KeY takes all this into account.

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Using Method Contracts

Expanding Method Bodies

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Disadvantages:

- Non-modular
- Duplication of effort
- Can lead to huge proofs

Expanding Method Bodies

$$\text{methodBodyExpand}\ \frac{\Gamma \Longrightarrow \mathcal{U}\langle \pi\ \texttt{method-frame(C,o)\{b\}}\ \omega\rangle\phi,\ \Delta}{\Gamma \Longrightarrow \mathcal{U}\langle \pi\ \texttt{o.m(p}_1\texttt{, ..., p}_n\texttt{)@C;}\ \omega\rangle\phi,\ \Delta}$$

Disadvantages:

- Non-modular
- Duplication of effort
- Can lead to huge proofs
- Sometimes the method body is not available (e.g. `native` methods)

Use Method Contract Rule

Given a contract (Pre, $Post$, $\mathcal{V}$):

Use Method Contract Rule

Given a contract $(Pre,\ Post,\ \mathcal{V})$:

$$\frac{}{\Gamma \texttt{ ==> } \mathcal{U}\langle \pi\ \texttt{o.m(p}_1\texttt{, ..., p}_n\texttt{)@C;}\ \omega\rangle \phi,\ \Delta}$$

Use Method Contract Rule

Given a contract $(Pre,\ Post,\ \mathcal{V})$:

$$\frac{\Gamma \Longrightarrow \mathcal{U}\,Pre,\ \Delta \qquad \text{(Pre)}}{\Gamma \Longrightarrow \mathcal{U}\langle \pi\ \texttt{o.m(p}_1\texttt{, ..., p}_n\texttt{)@C;}\ \omega\rangle\phi,\ \Delta}$$

Use Method Contract Rule

Given a contract $(Pre,\ Post,\ \mathcal{V})$:

$$\frac{\Gamma \texttt{ ==> } \mathcal{U}Pre,\ \Delta \qquad \Gamma \texttt{ ==> } \mathcal{U}\mathcal{V}(\texttt{exc} \doteq \texttt{null} \land Post \rightarrow \langle \pi\ \omega \rangle \phi),\ \Delta}{\Gamma \texttt{ ==> } \mathcal{U}\langle \pi\ \texttt{o.m(}\texttt{p}_1\texttt{, ..., p}_n\texttt{)@C;}\ \omega \rangle \phi,\ \Delta}$$

(Pre) (Post)

Use Method Contract Rule

Given a contract $(Pre,\ Post,\ \mathcal{V})$:

$$\frac{\begin{array}{ll}\Gamma \texttt{ ==> } \mathcal{U}Pre,\ \Delta & \text{(Pre)}\\ \Gamma \texttt{ ==> } \mathcal{U}\mathcal{V}(\texttt{exc} \doteq \texttt{null} \land Post \rightarrow \langle \pi\ \omega \rangle \phi),\ \Delta & \text{(Post)}\\ \Gamma \texttt{ ==> } \mathcal{U}\mathcal{V}(\texttt{exc} \neq \texttt{null} \land Post \rightarrow \langle \pi\ \texttt{throw exc; } \omega \rangle \phi),\ \Delta & \text{(Exc.)}\end{array}}{\Gamma \texttt{ ==> } \mathcal{U}\langle \pi\ \texttt{o.m(p}_1\texttt{, ..., p}_n\texttt{)@C; } \omega \rangle \phi,\ \Delta}$$

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END